2011 #5

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dw}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, t is measured in years from the start of 2010.

a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 ($t = ¼$).

b) Find $\frac{d^2w}{dt^2}$ in terms of W. Use $\frac{d^2w}{dt^2}$ to determine whether your answer in part a is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = ¼$.

c) Find the particular solution W=W(t) to the differential equation $\frac{dw}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

What you'll Learn About

- How to use tangent line approximations to estimate a functions value for a given value of x

Eulers Method
- Tangent Lines

$y = y_1 + \frac{dy}{dx}(x - x_1)$

1. Consider the differential equation $\boxed{\frac{dy}{dx} = x - 1}$ (slope). Let y = f(x) be the particular solution to the given differential equation with initial condition $\boxed{f(1) = 2}$. Use Euler's Method, starting at x = 1 with three steps of equal size, to approximate f(1.3). Show the work that leads to your answer.

$(1,2)$ $\frac{dy}{dx} = 0$ $y = 2 + 0(x-1)$ $\boxed{y = 2}$

$(1.1, 2)$ $\frac{dy}{dx} = .1$ $y = 2 + .1(x - 1.1)$

$y = 2 + .1(1.2 - 1.1)$

$(1.2, 2.01)$ $\frac{dy}{dx} = .2$ $y = 2.01 + .2(x - 1.2)$

$y = 2.01 + .2(1.3 - 1.2)$

$\boxed{(1.3, 2.03)}$ $\boxed{f(1.3) = 2.03}$

$2.02 - 1.8 = .22$

2. Consider the differential equation $\boxed{\frac{dy}{dx} = x - 2y}$. Let y = f(x) be the particular solution to the given differential equation with initial condition f(2) = 1. Use Euler's Method, starting at x = 2 with three steps of equal size, to approximate f(1.7). Show the work that leads to your answer.

$(2,1)$ $\frac{dy}{dx} = 2 - 2(1)$ $\frac{dy}{dx} = 0$ $y = 1 + 0(x-2)$ $y = 1$

$(1.9, 1)$ $\frac{dy}{dx} = 1.9 - 2 = -.1$ $y = 1 - .1(x - 1.9)$

$y = 1 - .1(1.8 - 1.9)$

$(1.8, 1.01)$ $\frac{dy}{dx} = 1.8 - 2(1.01) = -.22$ $y = 1.01 - .22(x - 1.8)$

$\boxed{(1.7, 1.032)}$

$y = 1.01 - .22(1.7 - 1.8)$ FR
$= 1.01 + .022$
$= 1.032$ MC